

Looking at Mathematical Proof – through questions and problems

1. What do you understand about the connection between “truth” and “proof” in general? *Could extend this to belief*

If an English teacher could only do 1 author – who? Shakespeare? How would I prove it?

“I believe the Beatles” How could I prove it?

Why is proof chosen? Mathematicians view proof in a more rigorous way than other disciplines?

2. What do you understand about the connection between “truth” and “proof” in Mathematics?

3. What do you understand to be the differences between deductive proof (or reasoning) and inductive proof (or reasoning)?

4. In the space below, give a simple example of a mathematical deductive proof.

5. Try proving the following by mathematical induction:

$$(a) 1 + 2 + 3 + 4 + 5 + \dots + (n - 1) + n = \frac{1}{2}n(n + 1)$$

(b) $1^2 + 2^2 + 3^2 + 4^2 + \dots + (n-1)^2 + n^2 = \frac{n(n+1)(2n+1)}{6}$

(c) $4^n + 2$ is divisible by 3 for all n

6. Use the Proof by *reductio ad absurdum* method to prove that $\sqrt{2}$ is irrational

Reducing something to absurd.

$\sqrt{2}$ irrational. — to prove this is to try to prove the opposite.

Assume $\sqrt{2}$ is rational

7. What do you understand by the term "Proof by Exhaustion"? How/when might it be suitable in Mathematics?

8. What do you know about visual proof in mathematics? Give an example of when it is used. ^{what is it?} Why might it be good?

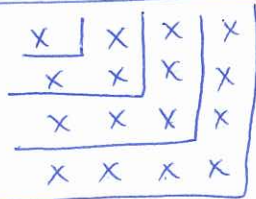
Maths is all about patterns.

$1 = 1$ $1 + 3 = 4$ $1 + 3 + 5 = 9$ $1 + 3 + 5 + 7 = 16$
(square numbers) If I add n odd numbers, I'll get
a square number = Mr Davis' theory. How could it be
proved? Using algebra? $S = \frac{n}{2} [2 + (n-1)2]$ a.p.s. arithmetic progression

$$= \frac{n}{2} [2 + 2n - 2]$$

$$= \frac{n}{2} (2n) = n^2 \quad \text{This is the proof.}$$

Visual proof



← no weaknesses, no opinion
Will it really go on forever?
Will it ALWAYS work?

9. A theorem I just invented – every odd number bigger than 1 can be written as the sum of a prime number and a power of 2.

eg

$$7 = 2^2 + 3$$

$$21 = 2^4 + 5$$

Try other examples.

What makes this rule very difficult to prove?

What would make this rule very easy to disprove?

10. Can everything in Mathematics be proved (disproved)? Is it possible to prove two conflicting ideas simultaneously?